ULTIMATE

'O' LEVEL A.MATHS

FORMULA LIST

$a^2+b^2=c^2$

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THE A-MATHS CHEAT SHEET

1. Polynomials:

(1) Dividend = Divisor × Quotient + Remainder

Quotient
Divisor) *Dividend*
Remainder

In long division: Degree of Remainder < Degree of Divisor

(2) **Remainder Theorem**: When $\mathrm{f}(x)$ is divided by $(ax - b)$ then remainder $\mathrm{R} = \mathrm{f}\left(\frac{b}{a}\right)$

(3) **Factor Theorem**: If $(ax - b)$ is a factor of $\mathrm{f}(x)$, then $\mathrm{f}\left(\frac{b}{a}\right) = 0$

(4) $a^3 + b^3 = (a + b)\,(a^2 - ab + b^2)$

$a^3 - b^3 = (a - b)\,(a^2 + ab + b^2)$

2. Partial Fractions:

If the fraction is improper, use long division to express it as a (polynomial + proper fraction) first.

(1) $\dfrac{px + q}{(ax + b)(cx + d)} = \dfrac{A}{ax + b} + \dfrac{B}{cx + d}$

(Linear factors)

(2) $\dfrac{px^2 + qx + r}{(ax + b)(cx + d)^2} = \dfrac{A}{ax + b} + \dfrac{B}{cx + d} + \dfrac{C}{(cx + d)^2}$

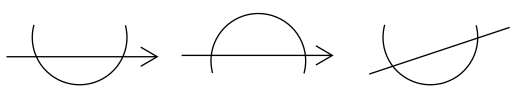
(Repeated factors)

(3) $\dfrac{px^2 + qx + r}{(ax + b)(x^2 + c^2)} = \dfrac{A}{ax + b} + \dfrac{Bx + C}{x^2 + c^2}$

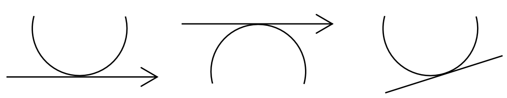
(Non-factorisable quadratic factor)

3. Discriminant, $D = b^2 - 4ac$:

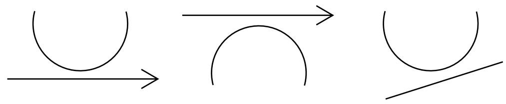
(1) $D > 0$ ⇔ 2 real & distinct roots ⇔ cuts x-axis at 2 points or line intersects curve at 2 points



(2) $D = 0$ ⇔ 2 real & equal roots ⇔ touches x-axis or line is a tangent to the curve



(3) $D < 0$ ⇔ No real roots ⇔ curve is entirely above x-axis (expression is always positive, $a > 0$, or always negative, $a < 0$) or line does not intersect curve



(4) $D \geq 0$ ⇔ Real roots ⇔ curve meets x-axis or line meets the curve

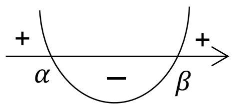
(5) $D < 0$ ⇔ expression is always negative or positive

4. Quadratic Inequalities:

If α and β are roots of $\mathrm{f}(x) = x^2 + bx + c = 0$, where $\beta > \alpha$, then

(i) $(x - \alpha)(x - \beta) > 0 \Rightarrow x < \alpha \text{ or } x > \beta$

(ii) $(x - \alpha)(x - \beta) \leq 0 \Rightarrow \alpha \leq x \leq \beta$



5. Quadratic Functions:

The coordinates of the **turning point** or the **vertex** of the graph of $y = a(x - h)^2 + k$ are (h, k).

- If $a > 0$, the minimum value of y is k, which occurs when $x = h$.
- If $a < 0$, the maximum value of y is k, which occurs when $x = h$.

6. Binomial Theorem:

(1) $(a + b)^n = a^n + \binom{n}{1} a^{n-1} b + \binom{n}{2} a^{n-2} b^2 + \ldots + \binom{n}{r} a^{n-r} b^r + \ldots + b^n$

where n is a positive integer and

$$\binom{n}{r} = \frac{n!}{r!\,(n - r)!} = \frac{n(n-1)\ldots(n-r+1)}{r!}$$

(2) General Term: $T_{r+1} = \binom{n}{r} a^{n-r} b^r$

(3) Useful results:

(i) $\binom{n}{1} = n$ (ii) $\binom{n}{2} = \dfrac{n(n-1)}{2!}$

(iii) $\binom{n}{3} = \dfrac{n(n-1)(n-2)}{3!}$

7. Surds:

(1) $\sqrt{a} \times \sqrt{b} = \sqrt{ab}$ (2) $\dfrac{\sqrt{a}}{\sqrt{b}} = \sqrt{\dfrac{a}{b}}$

(3) $\sqrt{a} \times \sqrt{a} = a$ (4) $m\sqrt{a} \pm n\sqrt{a} = (m \pm n)\sqrt{a}$

(5) $(\sqrt{a} + \sqrt{b})(\sqrt{a} - \sqrt{b}) = a - b$

(6) Rationalising of surds:

(i) $\dfrac{a}{\sqrt{b}} = \dfrac{a}{\sqrt{b}} \times \dfrac{\sqrt{b}}{\sqrt{b}} = \dfrac{a\sqrt{b}}{b}$

(ii) $\dfrac{a}{\sqrt{b} + \sqrt{c}} = \dfrac{a}{\sqrt{b} + \sqrt{c}} \times \dfrac{\sqrt{b} - \sqrt{c}}{\sqrt{b} - \sqrt{c}} = \dfrac{a(\sqrt{b} - \sqrt{c})}{b - c}$

8. Logarithms: For $y > 0, a > 0, b > 0, a \neq 1$:

(1) Definition: $\log_a y = x \Leftrightarrow a^x = y$

Special cases: (i) $\lg y = x \Leftrightarrow 10^x = y$

(ii) $\ln y = x \Leftrightarrow e^x = y$

(2) (i) $\log_a m + \log_a n = \log_a(mn)$

(ii) $\log_a m - \log_a n = \log_a\left(\dfrac{m}{n}\right)$

(iii) $\log_a m^n = n\log_a m$

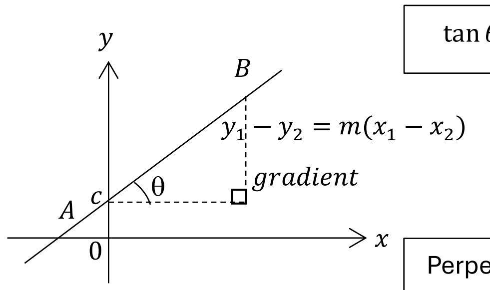
(iv) $\log_a 1 = 0$

(v) $\log_a a = 1$

(3) Change of Base: $\log_a b = \dfrac{\log_c b}{\log_c a}$

Special case: $\log_a b = \dfrac{1}{\log_b a}$

9. Coordinate Geometry



tan θ = gradient

Perpendicular Lines:

$m_1 m_2 = -1$

(1) Given $A(x_1, y_1)$ and $B(x_2, y_2)$,

(i) $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

(ii) Gradient of AB, $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

(iii) Midpoint of $AB = \left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

(iv) Equation of a straight line AB: $y - y_1 = m(x - x_1)$

(2) (i) Area of triangle $= \dfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_1 \\ y_1 & y_2 & y_3 & y_1 \end{vmatrix}$

(ii) Area of quadrilateral $= \dfrac{1}{2}\begin{vmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{vmatrix}$

Points need to be obtained in an **anti-clockwise** direction.

10. Equation of Circle

(1) Standard Form: $(x - a)^2 + (y - b)^2 = r^2$, centre $= (a, b)$ and radius $= r$ units

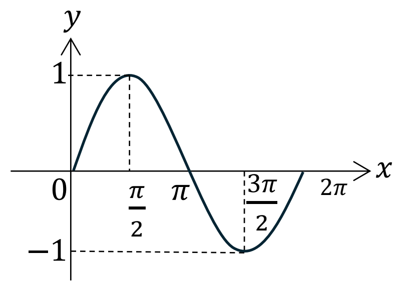
(2) General Form: $x^2 + y^2 + 2gx + 2fy + c = 0$,

centre $= (-g, -f)$ and radius $= \sqrt{f^2 + g^2 - c}$

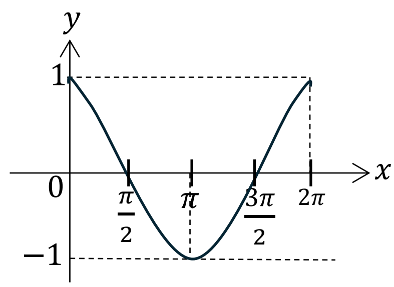
(3) The perpendicular bisector of any chord of a circle will pass through the centre of the circle.

11. Basic Trigonometric Graphs:

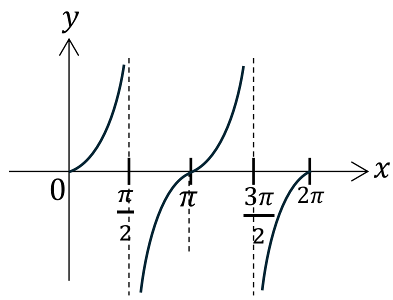
(1) $y = \sin x$



(2) $y = \cos x$



(3) $y = \tan x$



In general,

(i) For $y = a\sin bx \pm c$ and $y = a\cos bx \pm c$,

Amplitude $= |a|$, Period $= \dfrac{2\pi}{b}$

(ii) For $y = \tan bx \pm c$, Period $= \dfrac{\pi}{b}$

12. Trigonometric Identities & Formulae

(1) Reciprocal Identities: $cosec\ x = \frac{1}{\sin x}$, $\sec x = \frac{1}{\cos x}$,

$\cot x = \frac{1}{\tan x}$

(2) Quotient Identities: $\tan x = \frac{\sin x}{\cos x}$, $\cot x = \frac{\cos x}{\sin x}$

(3) Pythagorean Identities:

$$\sin^2 x + \cos^2 x = 1$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\cot^2 x + 1 = \text{cosec}^2\, x$$

(4) Addition Formulae:

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$$

(5) Double Angle Formulae:

$$\sin 2A = 2 \sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2 \tan A}{1 - \tan^2 A}$$

(6) Complementary Angles:

$\sin(90° - A) = \cos A$,

$\cos(90° - A) = \sin A$,

$\tan(90° - A) = \cot A$

(7) Supplementary Angles:

$\sin(180° - A) = \sin A$,

$\cos(180° - A) = -\cos A$,

$\tan(180° - A) = -\tan A$

(8) Negative Angles:

$\sin(-A) = -\sin A$, $\cos(-A) = \cos A$, $\tan(-A) = -\tan A$

(9) R – Formulae: $a \sin\theta \pm b\cos\theta = R\sin(\theta \pm \alpha)$

$a\cos\theta \pm b\sin\theta = R\cos(\theta \mp \alpha)$,

where $R = \sqrt{a^2 + b^2}$ and $\tan\alpha = \frac{b}{a}$

(10) Special Angles:

θ	0°	30°	45°	60°	90°	180°	270°	360°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	−1	0
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	−1	0	1
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞	0	∞	0

13. Differentiation Techniques:

(1) Given that n is a rational number and k is a constant,

(i) $\frac{d}{dx}[kf(x)] = k\frac{d}{dx}[f(x)] = kf'(x)$

(ii) $\frac{d}{dx}[f(x) \pm g(x)] = \frac{d}{dx}[f(x)] \pm \frac{d}{dx}[g(x)]$

(2) **Chain Rule**: $\frac{d}{dx}[f(x)]^n = n[f(x)]^{n-1}f'(x)$

(3) **Product Rule**:

$$\frac{d}{dx}[f(x) \cdot g(x)] = f(x) \cdot g'(x) + g(x) \cdot f'(x)$$

(4) **Quotient Rule**:

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

(5) **Derivatives of Algebraic Functions**:

Given that n is a rational number and a is a constant,

$$\frac{d}{dx}(ax^n) = anx^{n-1}$$

Special Cases: (i) $\frac{d}{dx}(ax) = a$

(ii) $\frac{d}{dx}(a) = 0$

(6) **Derivatives of Trigonometric Functions:**

Given that n is a rational number, a and b are constants,

(a) (i) $\frac{d}{dx}(\sin x) = \cos x$

(ii) $\frac{d}{dx}(\cos x) = -\sin x$

(iii) $\frac{d}{dx}(\tan x) = \sec^2 x$

(b) (i) $\frac{d}{dx}[(\sin(ax+b)] = a\cos(ax+b)$

(ii) $\frac{d}{dx}[(\cos(ax+b)] = -a\sin(ax+b)$

(iii) $\frac{d}{dx}[(\tan(ax+b)] = a\sec^2(ax+b)$

(c) (i) $\frac{d}{dx}(\sin^n x) = n\sin^{n-1}x\ \cos x$

(ii) $\frac{d}{dx}(\cos^n x) = -n\cos^{n-1}x\ \sin x$

(iii) $\frac{d}{dx}(\tan^n x) = n\tan^{n-1}x\ \sec^2 x$

(7) **Derivatives of Exponential Functions:**

Given that n is a rational number and a is a constant,

$$\frac{d}{dx}\left[e^{f(x)}\right] = f'(x)e^{f(x)}$$

Special cases: (i) $\frac{d}{dx}[e^x] = e^x$

(ii) $\frac{d}{dx}[e^{ax+b}] = ae^{ax+b}$

(8) **Derivatives of Logarithmic Functions:**

$$\frac{\mathrm{d}}{\mathrm{d}x}[\ln \mathrm{f}(x)] = \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}$$

Special cases: (i) $\frac{\mathrm{d}}{\mathrm{d}x}[\ln x] = \frac{1}{x}$

(ii) $\frac{\mathrm{d}}{\mathrm{d}x}[\ln(ax+b)] = \frac{a}{ax+b}$

14. Increasing & Decreasing Functions:

(i) y is an increasing function $\Leftrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} > 0$

y is an decreasing function $\Leftrightarrow \frac{\mathrm{d}y}{\mathrm{d}x} < 0$

(ii) $\frac{\mathrm{d}y}{\mathrm{d}x}$ is an increasing function $\Leftrightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} > 0$

$\frac{\mathrm{d}y}{\mathrm{d}x}$ is an decreasing function $\Leftrightarrow \frac{\mathrm{d}^2y}{\mathrm{d}x^2} < 0$

15. Applications of Differentiation:

(i) Gradient Function & Tangent:

Gradient of curve at A

= Gradient of tangent at A

$= Value\ of\ \frac{\mathrm{d}y}{\mathrm{d}x}$ at A

(ii) Rates of Change: $\frac{\mathrm{d}y}{\mathrm{d}t} = \frac{\mathrm{d}y}{\mathrm{d}x} \times \frac{\mathrm{d}x}{\mathrm{d}t}$

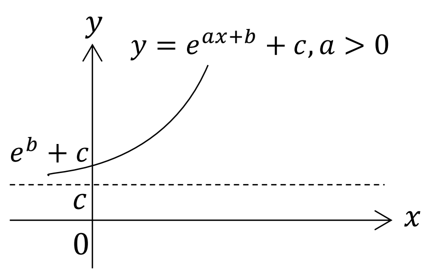
(iii) Stationary points/ Problems of Maxima & Minima:

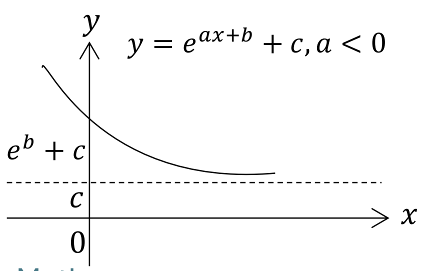
For Stationary points or maximum/minimum y,

$\frac{\mathrm{d}y}{\mathrm{d}x} = 0$

Checking of nature of stationary points: 1st or 2nd derivative test

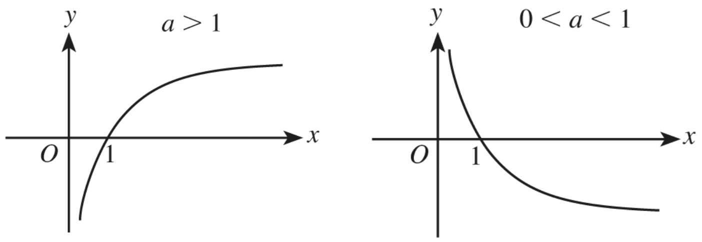
16. Graphs of Exponential Functions:





17. Graphs of Logarithmic Functions:

$y = \log_a x$, where $a > 0$ and $a \neq 1$



18. Integration

(1) Indefinite integral:

$$\frac{\mathrm{d}}{\mathrm{d}x}[\mathrm{f}(x)] = \mathrm{f}'(x) \Leftrightarrow \int \mathrm{f}'(x)dx = \mathrm{f}(x) + C,$$

where C is an arbitrary constant

(2) Definite integral:

$$\int_a^b \mathrm{f}(x)\,\mathrm{d}x = [\mathrm{F}(x)]_a^b = \mathrm{F}(b) - \mathrm{F}(a)$$

(3) (i) $\int_a^a \mathrm{f}(x)\,\mathrm{d}x = 0$

(ii) $\int_a^b \mathrm{f}(x)\,\mathrm{d}x = -\int_b^a \mathrm{f}(x)\mathrm{d}x$

(iii) $\int_a^b \mathrm{f}(x)\,\mathrm{d}x + \int_b^c \mathrm{f}(x)\,\mathrm{d}x = \int_a^c \mathrm{f}(x)\,\mathrm{d}x$

19. Integration Techniques

1. Integration of ax^n and $(ax+b)^n$:

Given that n is a rational number, a and b are constants,

(i) $\int ax^n\,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + C$

Special Case: $\int a\,\mathrm{d}x = ax + C$

(ii) $\int (ax+b)^n\,\mathrm{d}x = \frac{(ax+b)^{n+1}}{a(n+1)} + C$

2. Integration of Trigonometric Functions:

(1) (i) $\int \cos x\,\mathrm{d}x = \sin x + C$

(ii) $\int \sin x\,\mathrm{d}x = -\cos x + C$

(iii) $\int \sec^2 x\,\mathrm{d}x = \tan x + C$

(2) (i) $\int \cos(ax+b)\,\mathrm{d}x = \frac{1}{a}\sin(ax+b) + C$

(ii) $\int \sin(ax+b)\,\mathrm{d}x = -\frac{1}{a}\cos(ax+b) + C$

(iii) $\int \sec^2(ax+b)\,\mathrm{d}x = \frac{1}{a}\tan(ax+b) + C$

(3) (i) $\int \cos^2 x \, \mathrm{d}x = \int \frac{1}{2}(1 + \cos 2x)\mathrm{d}x$

$= \frac{1}{2}\left(x + \frac{1}{2}\sin 2x\right) + C$

(ii) $\int \sin^2 x \, \mathrm{d}x = \int \frac{1}{2}(1 - \cos 2x)\mathrm{d}x$

$= \frac{1}{2}\left(x - \frac{1}{2}\sin 2x\right) + C$

(iii) $\int \tan^2 x \, \mathrm{d}x = \int (\sec^2 x - 1)\, \mathrm{d}x$

$= \tan x - x + C$

<u>3. Integration of Exponential Functions:</u>

$$\int e^{ax+b}\mathrm{d}x = \frac{1}{a}e^{ax+b} + C$$

Special Case: $\int e^x \mathrm{d}x = e^x + C$

<u>4. Integration of $(ax+b)^{-1}$:</u>

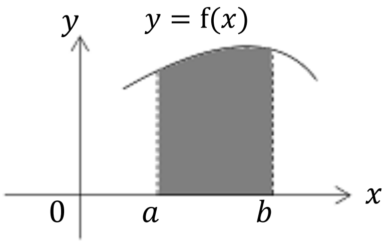
$$\int (ax+b)^{-1}\, \mathrm{d}x = \int \frac{1}{ax+b}\, \mathrm{d}x = \frac{1}{a}\ln(ax+b) + C$$

Special Case: $\int x^{-1}\, \mathrm{d}x = \int \frac{1}{x}\, \mathrm{d}x = \ln x + C$

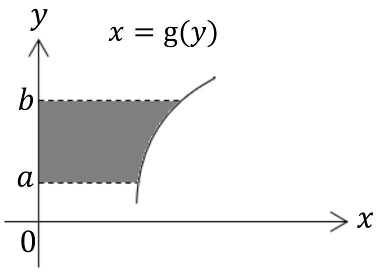
20. Applications of Integration:

<u>1. Area under a curve:</u>

(i) Area bounded by a curve and axes:

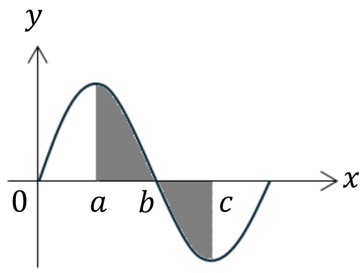


$$A = \int_a^b y\, \mathrm{d}x = \int_a^b \mathrm{f}(x)\, \mathrm{d}x$$



$$A = \int_a^b x\, \mathrm{d}y = \int_a^b \mathrm{g}(y)\, \mathrm{d}y$$

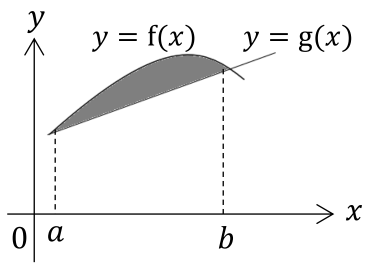
(ii) Negative area:



Since $\int_a^b \mathrm{f}(x) > 0$ and $\int_b^c \mathrm{f}(x) < 0$

$$A = \int_a^b \mathrm{f}(x) + \left|\int_b^c \mathrm{f}(x)\mathrm{d}x\right|$$

(iii) Area between a curve and line



$$A = \int_a^b \mathrm{f}(x)\mathrm{d}x - \int_a^b \mathrm{g}(x)\mathrm{d}x$$

$$= \int_a^b [\mathrm{f}(x) - \mathrm{g}(x)]\mathrm{d}x$$

<u>2. Kinematics:</u>

(1) $s \to v = \frac{\mathrm{d}s}{\mathrm{d}t} \to a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{\mathrm{d}^2 s}{\mathrm{d}t^2}$

$s = \int v\, \mathrm{d}t \leftarrow v = \int a\, \mathrm{d}t \leftarrow a$

(2) (i) Instantaneous rest: $v = 0$

(ii) Max or Min velocity: $a = 0$

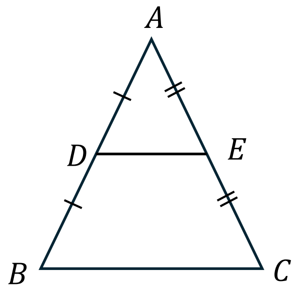
(iii) Returns to reference point O: $s = 0$

21. Plane Geometry:

(1) Midpoint Theorem:

In ΔABC, D and E are midpoints of AB and AC respectively.

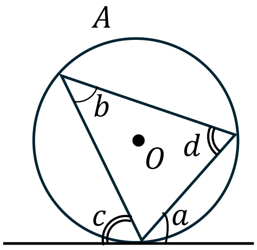
Then, (i) DE is parallel to BC, and (ii) $DE = \frac{1}{2}BC$



(2) Alternate Segment Theorem (or Tangent-Chord Theorem)

The angle between the tangent and the chord at the point of contact is equal to the angle subtended by a chord in the alternate segment.

$\angle a = \angle b$ and $\angle c = \angle d$



22. Linear Law

To convert a non-linear equation involving x and y into the linear form, express the equation in the form of $Y = mX + c$, where X and Y are expressions in x and/or y.

- The variables X and Y must contain only the original variables x and/or y, but they must not contain the original unknown constants such as a and b.
- The constants m and c must contain only the original unknown constants such as a and/or b, but they must not contain the original variables x and y.